Rayleigh scattering limits for low-level bidirectional reflectance distribution function measurements: corrigendum

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In previous research [C. C. Asmail *et al*. Appl. Opt. **33,** 6084–6091 (1994)] an estimate was given of the low-level bidirectional reflectance distribution function (BRDF) limit due to Rayleigh scattering from the air molecules within the detector field of view. Although the underlying model was correct, a fault in the derivation led to a conclusion that contains an erroneous angular factor. A cosine factor in the equivalent BRDF derived by Asmail *et al*. [Appl. Opt. **33,** 6084–6091 (1994)], which was considered unphysical in that treatment, is incorrect and can obscure the correction in certain circumstances. The treatment below calculates the scattered flux from the gas molecules in the field of view and compares it with the flux scattered from a sample in the same incident beam.

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1. Introduction

The objective of the original paper[1] was use the Rayleigh scattering theory to model the equivalent bidirectional reflectance distribution function[2] (BRDF) from the gas molecules in the atmosphere contained within the field of view of the detector used for the BRDF measurement. It has been determined experimentally[3] that for some detector field-of-view conditions, modern electronic detection systems may not impose the low-level detection limit for BRDF measurements but that instead Rayleigh scattering of the gas molecules within the field of view may be of larger magnitude than the detection system noise level. The equation derived[1] for the equivalent BRDF contains a $\cos^{-1} \eta_0$ term that results in a singularity at normal of the incident beam. The angle η_0 was given as the angle of the observation measured from the direction of the incident flux. There is no physical reason for the equivalent BRDF to grow without bound when the detector is viewing normal to the incident beam in the absence of a sample. This inconsistency was recognized before publication and managed artificially by implementation of the so-called cosine-corrected BRDF. The analysis in the original paper formulated the equivalent BRDF contribution from the gas molecules from the definition

$$f = L_s/E_I,$$

where L_s is the scattered radiance and E_I is the incident irradiance on the sample. The radiance from the sample gas was incorrectly calculated in the original paper. We believe a more direct method is shown below.

2. Correction

The measurement of the equivalent BRDF is associated with the measurement of a reference sample with a BRDF that we designate f_R. The following analysis pertains to the geometry presented in Ref. 1. To obtain the equivalent BRDF, f, we estimate the flux of scattered light reaching the detector position, p, with and without the reference sample. In vacuum, the scattered flux Φ_R at the detector when a sample with a BRDF of f_R is exposed to an average irradiance given by the incident flux Φ_I divided by the area irradiated is

$$\Phi_R = f_R \Phi_I \Omega \cos \theta_R, \tag{1}$$

where Ω is the collection solid angle A_p/r^2. A_p is the area of the detector at a distance r from the sample, and the dimensions of the detector are assumed small

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compared with r, and hence the correction for equivalent BRDF is only valid for the circumstance in which the collection solid angle is small. The angle of the viewed scattered radiation, θ_R, is measured from the sample normal as in the usual BRDF measurement arrangement.

The flux at p from vertically polarized light when there is no sample present is taken from Eq. (3) in Ref. 1:

$$\Phi_{\mathrm{Gas}} = A_p l N \frac{k^4|\alpha|^2 \Phi_I}{r^2}, \tag{2}$$

where N and α are the number density and the polarizability of the molecules in the volume of the gas seen by the detector, which was given by the quantity $(A_d l)$ in Ref. 1. The cross-sectional area of the incident light beam is A_d, and l is the length of the beam seen by the detector at point p. The quantity k is the wave number of the incident radiation, $2\pi/\lambda$. The incident flux Φ_I was represented in terms of irradiance and area of the beam in Ref. 1. The ratio of these fluxes with and without the sample is then

$$\frac{\Phi_R}{\Phi_{\mathrm{Gas}}} = \frac{f_R \cos\theta_R}{lNk^4|\alpha|^2}. \tag{3}$$

The ratio of the fluxes should in turn be equal to the ratio of the BRDF's of the sample and the volume of gas, provided the geometry of the apparatus is the same for both circumstances. The numerator is the BRDF of the reference sample scaled by the cosine of the viewing angle of the scattered light with respect to the sample normal. The denominator can be taken to be the effective BRDF of the volume of the gas molecules contained within the detector field of view, that is,

$$f = lNk^4|\alpha|^2. \tag{4}$$

This is the same expression as that derived in Ref. 1 with the exception of the cosine η_0 term. Although the BRDF of a reference sample was used in the derivation, there is no need to carry out a BRDF measurement relative to a reference sample during calculation of the effective BRDF from Eq. 4. One can also obtain the effective BRDF by use of Eq. (2), accounting properly for the solid angle and utilizing the definition of BRDF from Eq. (1). This expression can be used to estimate the noise floor and to make corrections owing to the systematic errors caused by Rayleigh scattering in low level BRDF measurements.

3. Conclusions

We hope that this correction does not cause great inconvenience, and we apologize for the error in Ref. 1 that may have confused the issue of estimating noise floors. The correction is more intuitively appealing than that derived earlier, which required an artificial correction to avoid an unphysical angular dependency. Further research needs to be done on this topic to account for any scatter in the light beam after its interaction with the surface and to account for any effects on the polarization of the signal.[4,5] Some experimental circumstances may make the approximations and the expression for Rayleigh scattering used here invalid and thus require a more detailed modeling to account for the effects of Rayleigh scattering in a particular instrument. A more complete treatment of this topic for the general scattering case will be the topic of future efforts by the National Institute of Standards and Technology.

References and Notes

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2. The BRDF caused by instrument limitations when there is no sample scatter present is called the equivalent BRDF or the instrument signature.
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